Elementary functions- Graphs

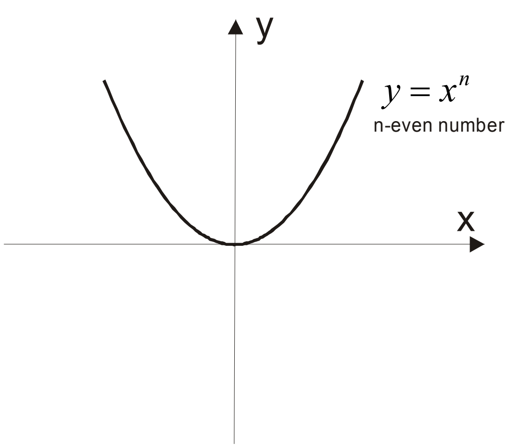
Basic elementary functions are:

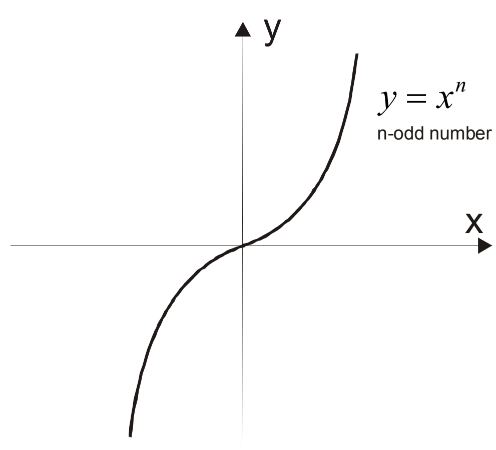
- Constant function
- The degree function
- Exponential function
- Logarithmic functions
- Trigonometric functions
- Inverse trigonometric functions
- Hyperbolic functions

Elementary functions are called functions that can be set using the basic elementary functions and constants, using finitely many operations of addition, subtraction, multiplication, division and composition of basic elementary functions.

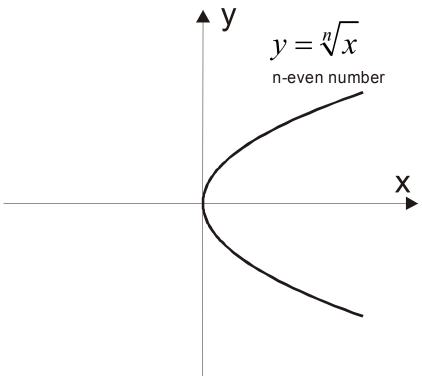
Note: This is not a strict definition of elementary functions.You learn that definition as say your professor…

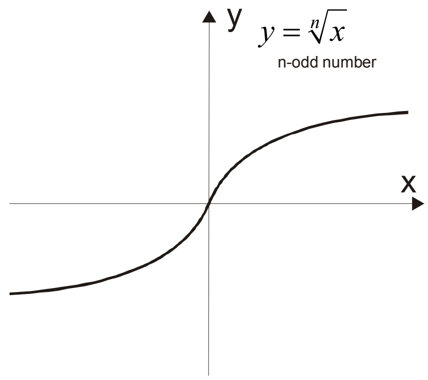
We are here to clarify a few things and remind you how look graphic…





These are graphs of degree functions where **exponent is a natural number**. All the graphics look like, except that, depending on degree they narrow or broad.

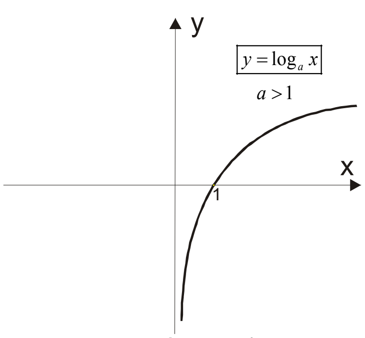




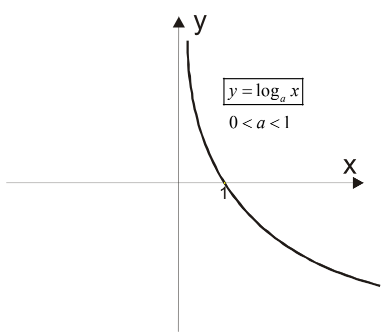
These are graphs of power functions where the **exponent is a rational number.**

We need to remember that $y=\sqrt[n]{x}$, where n is **an even number** is defined only for $x\in[0,\infty)$ that is $x\geq 0$, funkcija

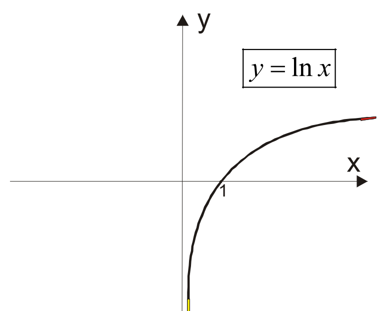
$y=\sqrt[n]{x}$ when n is **odd number** defined on the entire set R, that is $x\in(-\infty,\infty)$



picture 1



picture 2



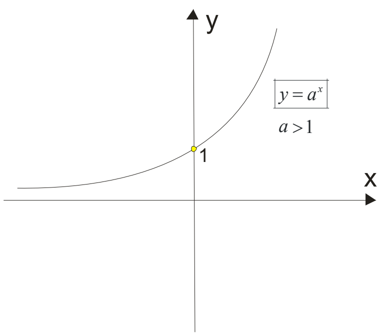
picture 3

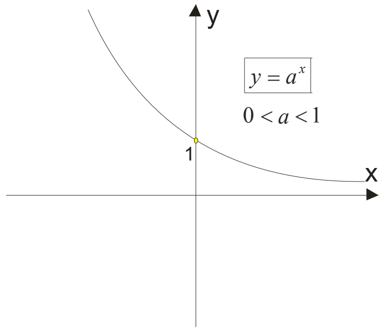
Logarithmic function is defined for $x>0$.

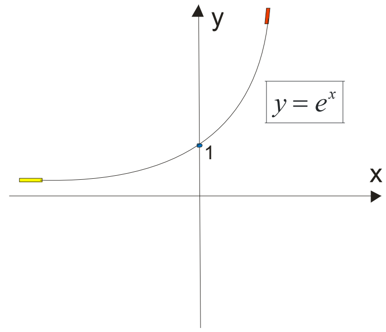
We say that $\ln 0=-\infty$. Now we can see that from elementary graphic (picture 3)

When x approaches 0 from the positive side function tends infinity (minus): $\lim\limits_{x\to 0+\varepsilon}\ln x=-\infty$

And we said $\ln\infty=\infty$.





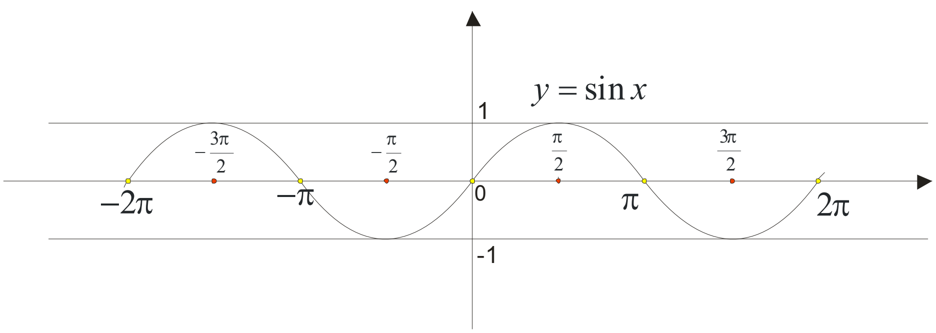


Važno je da su one svuda definisane: It is important that they are everywhere defined: $\forall x \in R$.

When we explained the limit, we said that $e^{-\infty} = 0$ and $e^{\infty} = \infty$

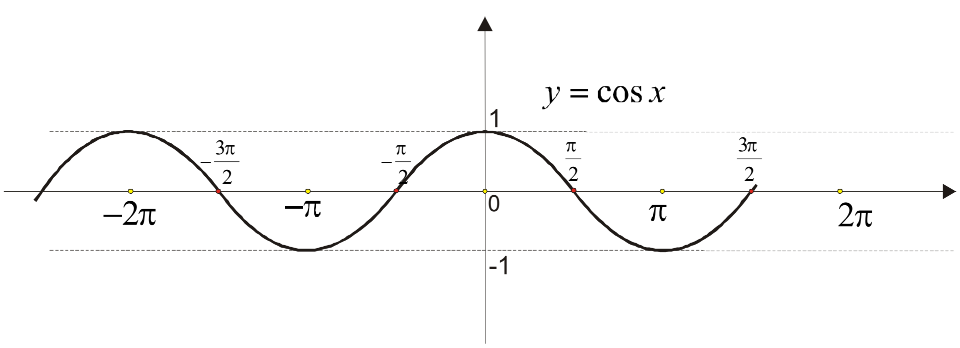
Trigonometric functions:

Sine function **y = sinx** is the basic trigonometric functions.

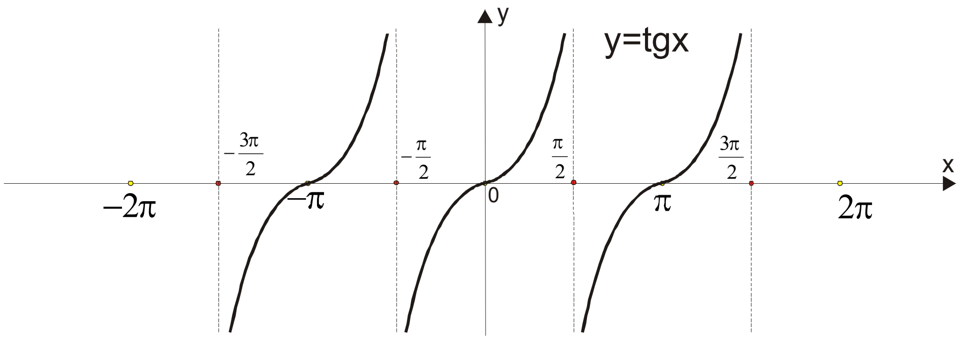


Other trigonometric functions defined as follows:

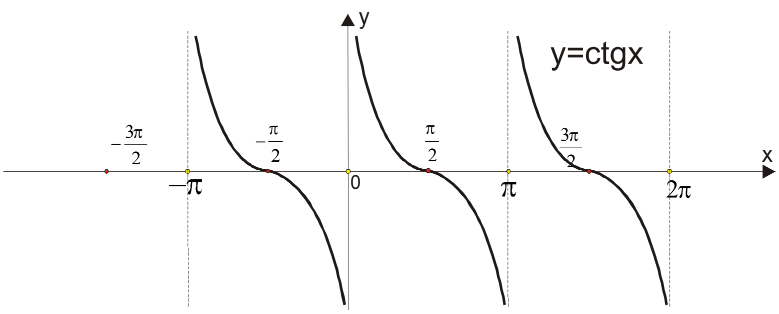
$$\cos x = \sin(x + \frac{\pi}{2})$$



$$tgx = \frac{\sin x}{\cos x}$$



$$ctgx = \frac{\cos x}{\sin x}$$

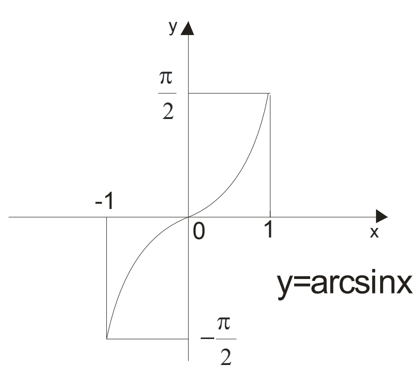


<u>Inverse trigonometric functions:</u>

i) Arcsine

Beware: the function y = sinx has no inverse function, because it is not a bijection!

But if we look at its restriction on the interval $[-\frac{\pi}{2}, \frac{\pi}{2}]$ then $f^{-1}: [-1,1] \to [-\frac{\pi}{2}, \frac{\pi}{2}]$ and



Still remember that:

$$\arcsin(\sin x) = x \quad \text{for } x \in [-\frac{\pi}{2}, \frac{\pi}{2}]$$
$$\sin(\arcsin x) = x \quad \text{for } x \in [-1,1]$$

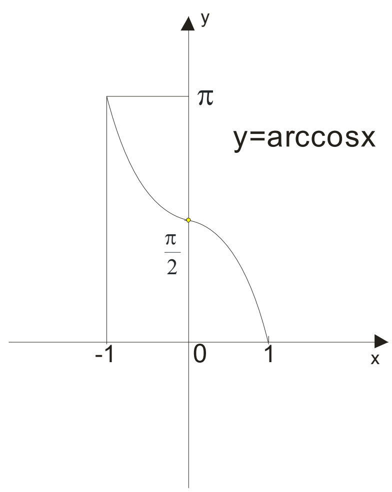
The function is defined for $x \in [-1,1]$

Zero function is in x=0.

ii) Arccosine

And here we will observe the same reason a restriction of **y = cos x** on the interval $[0, \pi]$**.**

$g^{-1} : [-1,1] \to [0, \pi]$



Valid:

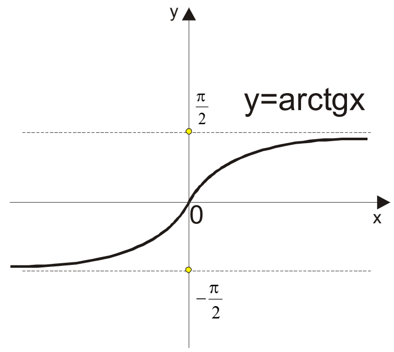
$$\arccos(\cos x) = x \quad \text{for } x \in [0, \pi]$$
$$\cos(\arccos x) = x \quad \text{for } x \in [-1,1]$$

The function is defined for $x \in [-1,1]$

Nula funkcije je u x =1

iii) Arctg

Looking at the restriction of $y = \text{tgx}$ on the interval $[-\frac{\pi}{2}, \frac{\pi}{2}]$ and $h^{-1} : R \to [-\frac{\pi}{2}, \frac{\pi}{2}]$



$arctg(tgx) = x$ for $x \in [-\frac{\pi}{2}, \frac{\pi}{2}]$

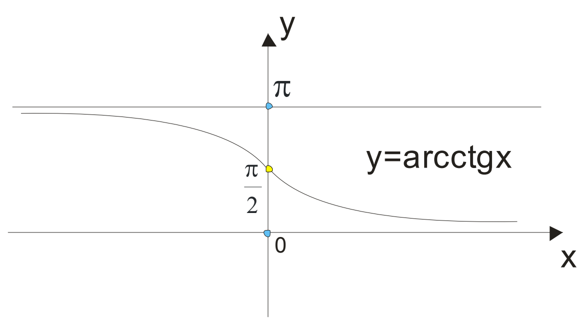
$tg(arctgx) = x$ for $x \in R$

Function is defined on the entire set of R.

Zero function is x = 0.

iv) Arcctg

$k^{-1} : R \to [0, \pi]$



$arcctg(ctgx) = x$ for $x \in [0, \pi]$

$ctg(arcctgx) = x$ for $x \in R$

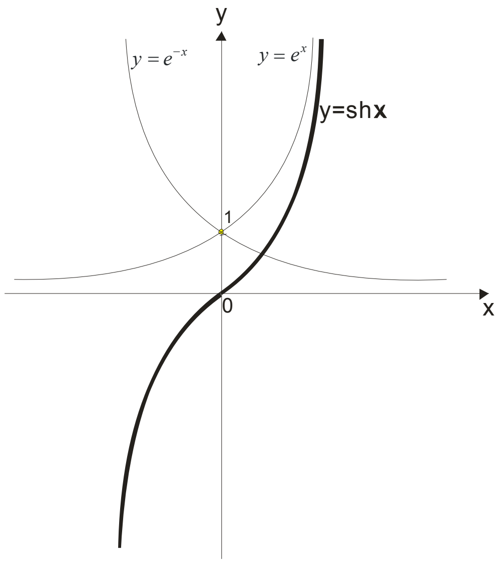
The function is everywhere defined. No zeros.

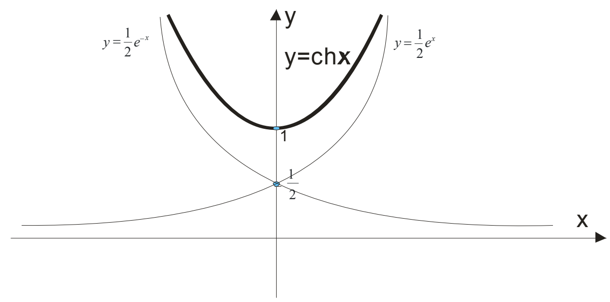
Hyperbolic functions

These are the functions: hyperbolic sine: $shx = \frac{e^x - e^{-x}}{2}$, hyperbolic cosine: $chx = \frac{e^x + e^{-x}}{2}$

hyperbolic tangent : $thx = \frac{e^x - e^{-x}}{e^x + e^{-x}}$ and hyperbolic cotangent: $cthx = \frac{e^x + e^{-x}}{e^x - e^{-x}}$

Graphs of these functions are obtained from graphics $y=e^{x}$ and $y=e^{-x}$ ($y=\frac{1}{2}e^{x}$ and $y=\frac{1}{2}e^{-x}$)





There are valid identities:

$$ch^2x - sh^2x = 1$$
$$sh(x+y) = shx \cdot chy + chx \cdot shy$$
$$ch(x+y) = chx \cdot chy + shx \cdot shy$$
$$sh2x = 2 \cdot shx \cdot chx$$
$$ch2x = ch^2x + sh^2x$$